## Discrete Mathematics: Combinatorics

Background

This is a continuation of MAT2612, dealing with counting, relations, functions and partially ordered sets.

**Graph theory**

* isomorphic graphs
* planar graphs
* Euler cycles
* Hamilton circuits
* graph colouring
* trees
* the travelling salesperson problem
* minimal spanning

**Combinatorics**

* basic counting principles
* generating functions
* recurrence relations
* inclusion-exclusion principle.

**Lesson 0**

Introduction to graphs

Simple Graphs are made of two types of objects:

(no loops, multiple edges or directed edges)

Edges - Size

the number of edges in the graph

Vertices - Nodes

Order

the number of vertices in the graph

Sets in this subject follow many of the same principles as ordinary sets (subsets, improper subsets, complimentary sets etc)

A picture containing watch

Description automatically generated[1] Ordered pair with some a finite vertex set V and some Edge set E

[2] Two graphs are equal if they have the same vertex and edge sets

A picture containing watch

Description automatically generated

A picture containing watch

Description automatically generated

**Lesson 0**

Adjacency

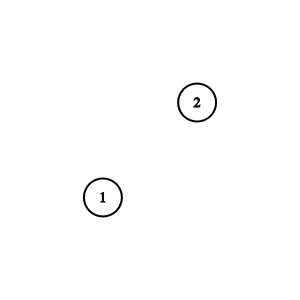
**Adjacent Vertices**

Two vertices are adjacent if they are joined by an edge

Let be a graph and , then and are adjacent in

Let be a graph, then are adjacent in there is an edge in joining a,b

Adjacent Non-adjacent

Diagram, shape, arrow

Description automatically generated

**Adjacent Edges**

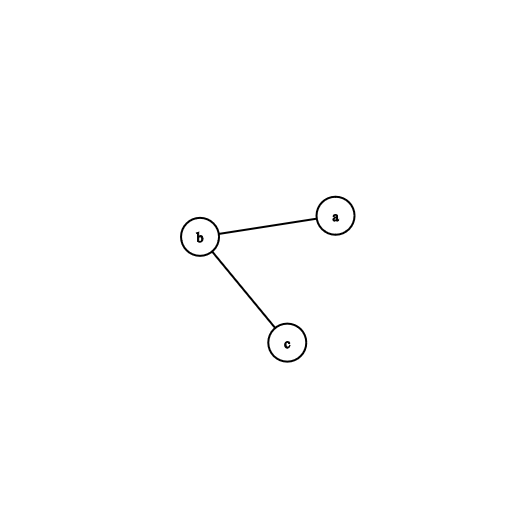
Two edges are adjacent if they are incident with a common vertex

Example

bc is incident with the vertex b

ab is incident with the vertex b

ab is adjacent to bc



Example

ab is adjacent to bc

bc is adjacent to cd

cd is adjacent to ad

ab is adjacent to bc

Shape

Description automatically generated

**Isolated Vertices**

A vertex is isolated if it not joined by any edge

**Lesson 0**

Degree of a Vertex

The degree of a vertex is the number of edges incident to it

*The number of vertices adjacent to it*

Example

a has a degree of 2

b has a degree of 1

c has a degree of 1

the minimum degree of G is 1

the maximum degree of G is 2

**Shape

Description automatically generated**

|  |  |
| --- | --- |
| **Degree of V** | **Name** |
| 0 | Isolated Vertex |
| 1 | End Vertex, Leaf |
| 2n | Even vertex |
| 2n+1 | Odd vertex |

**Theorem:** The sum of degree of all vertices of a graph is twice the size of graph.

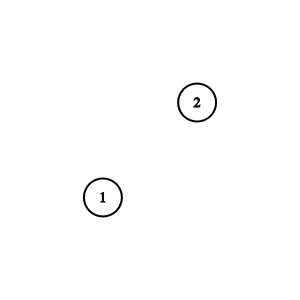
**Lesson 0**

The empty graph, trivial graph, and null graph

Example: empty graph

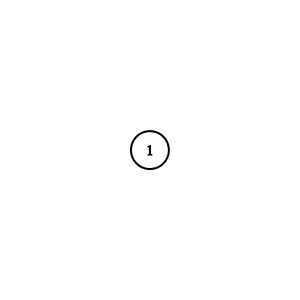
Empty Graph is any graph with some empty Edge set E

*The complement of the complete graph on 2 vertices*



Example: trivial graph

*The empty graph on 1 vertex*



Example: null graph

(Sometimes called the empty graph)

*No vertices. No edges. Is sometimes a graph, sometimes not*

**Lesson 0**

Subgraphs

* *the vertex set of H is the subset of vertex set of G AND*
* *the edge set of H is the subset of edge set of G*

Every graph is also a subgraph of itself (improper subgraph)

Example: Which of the graphs is a subgraph of another?

Diagram

Description automatically generated

Diagram

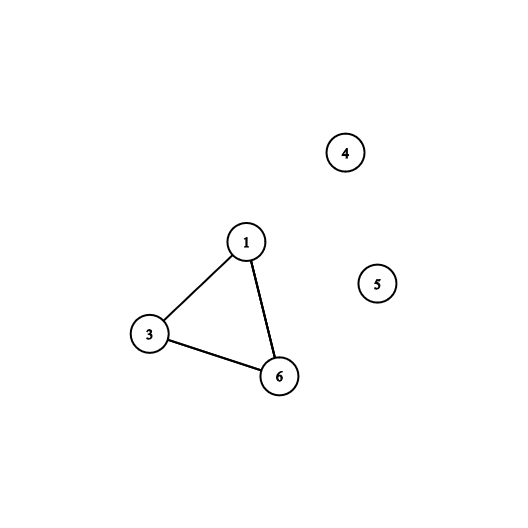
Description automatically generatedDiagram, shape

Description automatically generated

Spanning subgraph

* *the vertex set of H is equal to the subset of vertex set of G AND*
* *the edge set of H is the subset of edge set of G*

Example: Which of the graphs is a spanning subgraph of another?

Diagram

Description automatically generated

**Lesson 0**

Proper and Improper Subgraphs

Proper subset

Improper subset

**Lesson 1**

Degree sequence

This is essentially the process of listing the degrees of the vertices of a graph in a particular sequence.

Each graph has exactly one “non-increasing” degree sequence, that is, the sequence of degrees in descending order.

Let be a graph of order n.

Then, is a degree sequence of

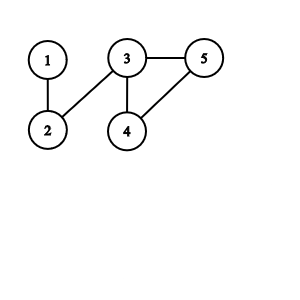
iff the vertices of can be labeled

such that for all

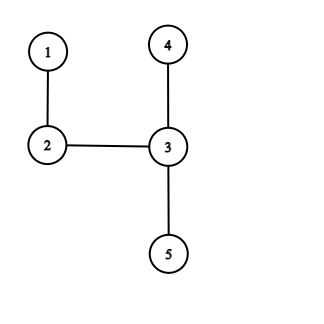
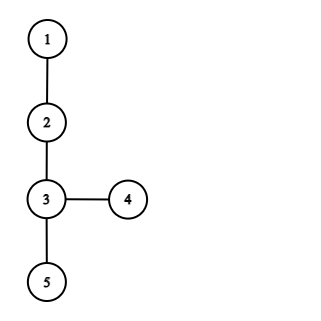
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1,2,3,2,2

3,2,2,2,1 Non-increasing degree sequence



If two graphs are isomorphic (same structure), then their degree sequences are the same



: 3,2,1,1,1 3,2,1,1,1

But if two graphs have the same degree sequences, they are not necessarily isomorphic

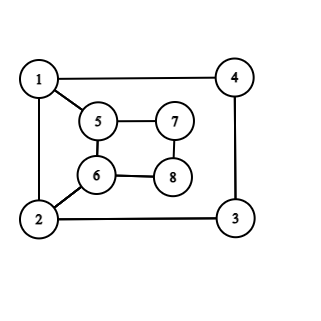
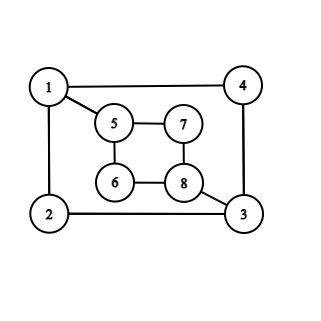
: 3,3,3,3,2,2,2,2 3,3,3,3,2,2,2,2

v5 has a degree of 3 v1 has a degree of 3

v5 has a degree of 3 v2 has a degree of 3

v5 has a degree of 3

v6 has a degree of 3



**Graphical degree sequences**

A finite sequence of non-negative integers is graphical if it is a degree sequence of some graph.

Example: some sequences which are NOT a degree sequence of some graph

3,3 *max no. of degrees in graph = no of vertices – 1*

1,1,1 *A graph must have an even number of odd vertices*

Example: ASS1 1b

Up to isomorphism, find all graphs with degree sequence (1, 1, 1, 1, 2, 2, 4)

**Lesson 2**

Compliment of a graph

Complement of a simple graph is a simple graph having

- All the vertices of .

- An edge between two vertices and iff there exists no edge between and in the original graph .

To rephrase the above, and put together form a complete graph .

<https://math.stackexchange.com/questions/1973586/graph-theory-question-finding-the-number-of-vertices-from-number-of-edges>

**Number of Vertices**

**Number of Edges**

Example: ASS1 Q1a

A graph has 10 vertices and 21 edges. Find the number of edges in the complement of .

- basic definitions & properties of graphs

(simple mathematical models of networks)

- modelling of real-life problems in terms of graphs

- isomorphism

- special types of graphs: connected graphs

- special types of graphs: bipartite graphs